

# Galactic Streams of Cosmic-ray Electrons and Positrons

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Isotropy is a key assumption in many models of cosmic-ray electrons and positrons. We find that simulation results imply a critical energy of  $\sim 10\text{--}1000$  GeV above which  $e^\pm$  can spend their entire lives in streams threading magnetic fields, due to energy losses. This would restrict the number of  $e^\pm$  sources contributing at Earth, likely leading to smooth  $e^\pm$  spectra, as is observed. For positrons, this could be as few as one, with an enhanced flux that would ease energetics concerns of a pulsar origin of the positron excess, or even zero, bringing dark matter into play. We conclude that ideas about  $e^\pm$  propagation based on either isotropic diffusion or turbulent fields must be changed.

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**Introduction.**— Energy losses due to synchrotron radiation and inverse-Compton scattering severely limit the distances over which cosmic-ray electrons and positrons can remain highly energetic [1, 2]. Thus, observations of these particles provide an opportunity to examine the recent high-energy history of within a few hundred parsecs of Earth. A number of experiments have contributed to the recent developments in this area [3–7] and more are poised to break into the multi-TeV regime [8–10]. The objects that may yield measurable fluxes in this range are few [11] and represent perhaps the most immediate hope of directly discovering an extrasolar cosmic-ray source.

Making use of this data requires understanding of the propagation of cosmic rays through magnetic fields after escaping their sources. Many standard treatments for electrons and positrons (e.g., Refs. [12–16]) assume isotropic diffusion. But why? In smooth fields, particles follow field lines, resulting in funneling in specific directions. As we will see, considering small scale turbulent variation in the field [17] alone ends up being too weak to cause isotropization for physically sensible assumptions in scenarios involving  $e^\pm$ .

We compare simulations of  $e^\pm$  propagation using random magnetic fields with such analytical solutions over a range of energies, distances, and times. We concentrate on particular realizations of fields, rather than the average over many such fields as implicitly assumed in diffusion models, since we can observe particles directly from only one location in the Galaxy. As an illustration, we show in Fig. 1 particle trajectories arising from nine simulated  $e^\pm$  sources distributed within one random field, as described in detail below. We explicitly see, similar to [18], that rather than being distributed in spherically symmetric diffusive distributions, the particles are initially confined to filamentary structures, which we refer to as “streams”.

Importantly, we take into account particle energy losses, which is crucial here, since cooling necessitates that electrons and positrons can only travel for a limited time with energies exceeding a given value, in stark contrast to the case of protons. We argue that, because of the

losses, the consequence of this approach is that electrons and positrons with measured energies  $\gtrsim 10\text{--}1000$  GeV can be expected to spend their entire lives within streams, *never reaching the isotropic diffusive regime*.

This containment of cosmic-ray electrons and positrons would be far reaching, including a need to abandon isotropic  $e^\pm$  diffusion calculations in this regime. We discuss how this limits the number of sources that can contribute to the measured  $e^\pm$  spectra, which has the advantage of accounting for the absence of features expected from the multitude of potential nearby sources. This would affect models for the positron excess, with implications for a dark matter origin, and we further address additional factors that should be examined.

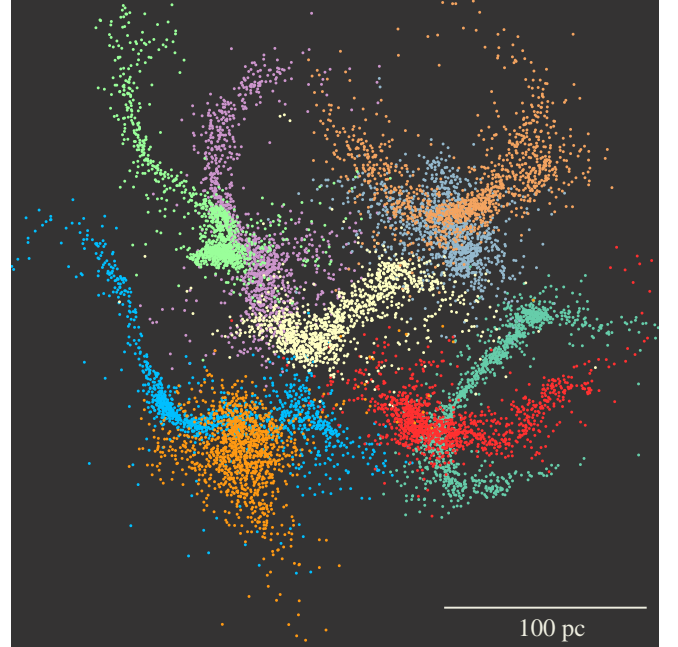


FIG. 1: Distributions of electrons with initial energies of 1 PeV after propagating 5000 yr in a  $3\mu\text{G}$  random magnetic field. Each stream (dots) corresponds to one of nine sources.

**Fields and streams.**— To study cosmic-ray electrons traversing a number of magnetic field realizations, we simulate the propagation of large numbers of particles to obtain particle densities over a range of times and distances. The Galactic field consists of a regular component with reversals in direction between spiral arms and a turbulent component with coherence length  $\sim 100$  pc [19]. The dominant field surrounding the Sun is tilted [20], with interpretations of the IBEX Ribbon yielding a  $\sim 3 \mu\text{G}$  field oriented  $\sim 55^\circ$  off of the Galactic plane [21, 22]. Fluctuations consistent with Kolmogorov turbulence are inferred over a wide range of scales [23]. Radio synchrotron data imply a local  $\sim 7.5 \mu\text{G}$  random field [24], and a few  $\mu\text{G}$  turbulent field may explain small-scale PeV proton anisotropies [25].

For a singly-charged particle within a magnetic field of strength  $B$ , the equation of motion can be written as

$$\frac{d\boldsymbol{\beta}}{dt} \simeq 0.925 \frac{\boldsymbol{\beta} \times \mathbf{B}}{E} \quad \boldsymbol{\beta} = \frac{d\mathbf{r}}{dt}, \quad (1)$$

where  $\boldsymbol{\beta}$  is a unit vector since the velocity is given in terms of  $c$ . Both distance and time are expressed in pc, energy in PeV, and  $B$  in  $\mu\text{G}$ , with a Larmor radius of

$$r_L \simeq 1.08 E/B. \quad (2)$$

We first follow the formulation laid out in Ref. [26] (see also [27–30]) to obtain a  $\sim 3 \mu\text{G}$  magnetic field  $\mathbf{B}_{k_0}^{k_1}$  between scales  $k_0$  and  $k_1$  whose power follows  $|B_k|^2 \propto k^{-(n+2)}$  ( $n=5/3$  for Kolmogorov turbulence), addressing other possibilities later. It is a practical impossibility to construct a magnetic field to the  $k$  values associated with the  $r_L$  of GeV–TeV particles, due to memory constraints. We instead reach smaller scales by using nested boxes,  $\mathbf{B}_{k_0}^{k_N}(\mathbf{r}) \propto \sum_{i=0}^N \eta^{-i/2} \mathbf{B}_{k_0}^{k_1}(\eta^i \mathbf{r})$ , where  $\eta = k_0/k_1$  and boxes repeat periodically when needed (similar to the method used in Ref. [31]). This maintains the proper normalization of power over all scales.

A sufficiently large  $k_0$  value ensures variation in the simulation box even in the largest scales and  $\eta$  can be as large as memory allows. We normalize the rms value of  $B_{k_0}^{k_N}$  to  $3 \mu\text{G}$  and choose a maximum length scale,  $l_{max} \propto 1/k_0$ , of 200 pc and a minimum scale,  $l_{min} \propto 1/k_N$ , smaller than  $r_L$ , with a coherence length  $l_c \simeq 40$  pc. Particles are injected isotropically at the source and Eq. (1) is solved via a fourth-order Runge-Kutta method with step sizes smaller by at least a factor of several than  $r_L$ .

Our interest is principally in high-energy electrons, for which energy losses arise from inverse-Compton (IC) scattering on ambient photons and synchrotron radiation. We approximate energy losses as continuous (IC is stochastic at very high energies [32]; however, gamma rays are not our focus), with  $-dE/dt = b(E) = b_0 E^2$ , so  $1/E = 1/E_g + b_0 t$ , where  $E_g$  is the energy at generation. Assuming a  $\sim 3 \mu\text{G}$  magnetic field and the cosmic microwave background (CMB),  $b_0 \simeq 5 \times 10^{-17} \text{ s}^{-1} \text{ GeV}^{-1}$ .

In Fig. 1, we show the positions of cosmic-ray  $e^\pm$  with  $E_g = 1 \text{ PeV}$  after propagating 5000 yr from sources at

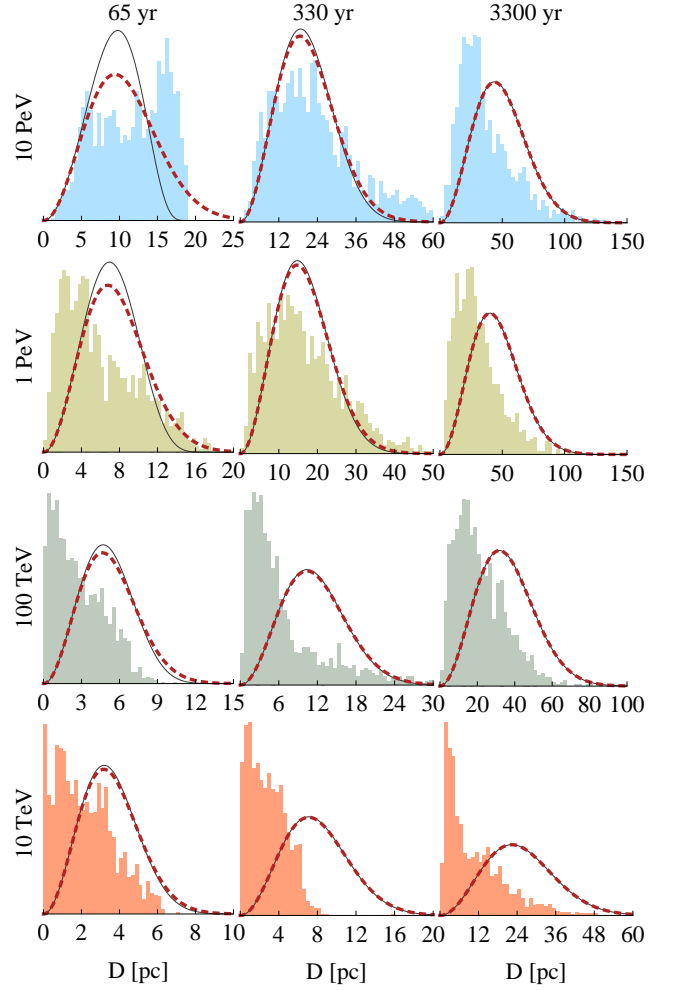


FIG. 2: Radial distributions of cosmic-ray  $e^\pm$  with initial energies of 10 TeV–10 PeV after propagation through a  $3 \mu\text{G}$  random magnetic field at given times (*bins*). Also shown are the expectations from isotropic diffusion (*dashed*) and Jüttner (*solid*) models, which are seen to poorly represent the data.

nine locations distributed within the same  $3 \mu\text{G}$  field configuration. We see that, rather than diffusing isotropically, the fluxes tend to escape along specific directions. This has been seen in the context of sources of Galactic protons with similar parameters [18].

Using the source position located at the center of Fig. 1, we consider instantaneous bursts for  $E_g$  covering 10 TeV–10 PeV (variable sources can be built up as combinations of successive bursts). We display in Fig. 2 snapshots of the radial distributions summed over all directions for these four energies at three given times.

**Comparison to analytical models.**— Assuming spherical symmetry, the diffusion equation yields a particle density  $n_d(r, t, E)$  from a bursting source [1] as

$$n_d(r, t, E) = \frac{e^{-r^2/r_{\text{dif}}^2}}{\pi^{3/2} r_{\text{dif}}^3} \frac{dN}{dE_g} \frac{dE_g}{dE}, \quad (3)$$

with  $\lambda(E, t) = \int_0^t dt' D[E(t')] = \int_{E_g}^E dE' D(E')/b(E')$  giving

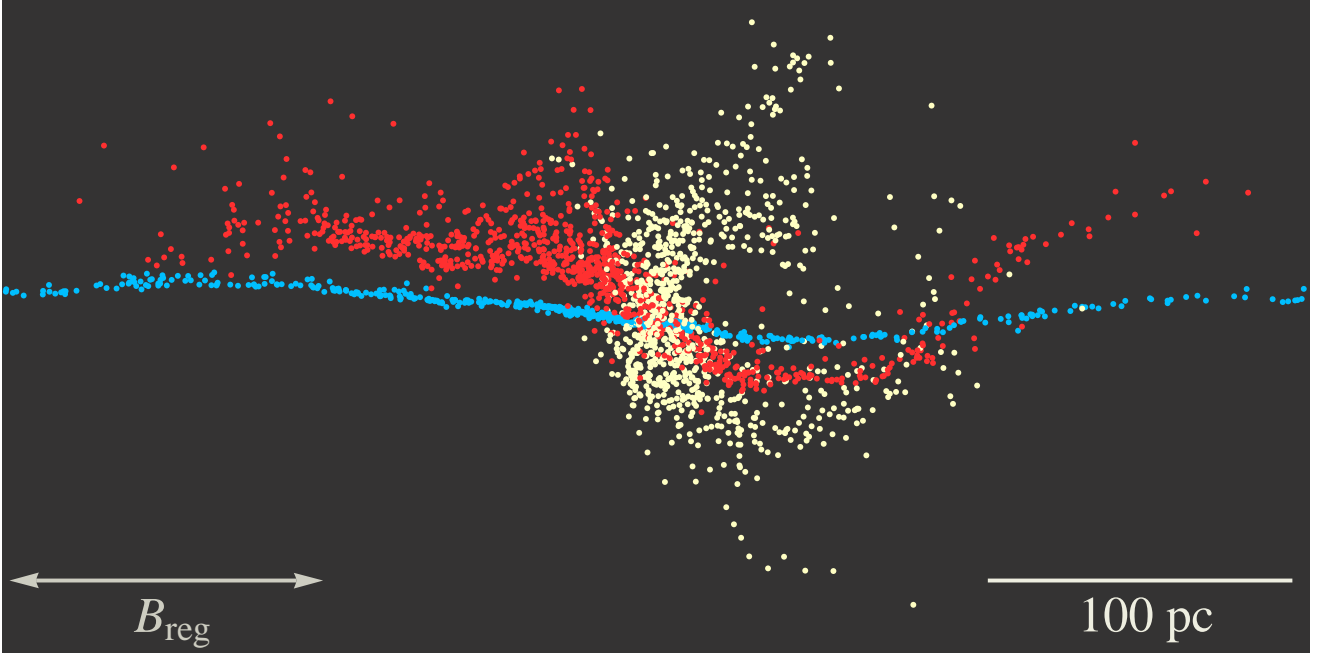


FIG. 3: Electron streams arising after propagating for 5000 yr with initial energies of 1 PeV. Here, we vary the ratio of regular to random field magnitudes as 0 (*white*), 1 (*red*), and 5 (*blue*), while fixing  $B_{\text{reg}} + B_{\text{rand}} = 3 \mu\text{G}$ . We see that increasing  $B_{\text{reg}}$  orients propagation along the regular field direction (as indicated).

$r_{\text{dif}}(E, t) = 2\sqrt{\lambda(E, t)}$ .  $E_g$  is mapped to the measured  $E$  after losses via  $dE_g/dE = (E_g/E)^2$ , using the above  $b_0$ .

Since the diffusive solution has been noted to fail at high energies [11, 33], we also consider a model based on the Jüttner particle distribution, which explicitly constrains  $v < c$  to eliminate superluminal behavior, giving

$$n_J(r, t, E) = \frac{\theta[1 - \xi] \alpha_J}{4\pi(ct)^3 K_1(\alpha_J)} \frac{e^{-\alpha_J/\sqrt{1-\xi^2}}}{(1 - \xi^2)^2} \frac{dN}{dE_g} \frac{dE_g}{dE}, \quad (4)$$

where  $\xi(r, t) = r/ct$ ,  $\theta$  is the step function,  $K_1$  is the modified Bessel function, and  $\alpha_J(E, t) = c^2 t^2 / (2\lambda(E, t))$  [33].

In Fig. 2, we compare the simulated densities to both models, using  $D(E) = 5.4 \times 10^{26} (E/\text{GeV})^{1/3} \text{cm}^2 \text{s}^{-1}$ , a value close to that seen in Ref. [18]. We note that this is lower by about an order of magnitude than used in phenomenological diffusion models [34]. Neither provide a good representation of the radial distributions, nor can they convey the inherent asymmetries (as exhibited in Fig. 1). The enhanced densities near the source position, resembling the expectation from diffusion in one dimension, and concentrations in flux as compared to isotropic propagation lead us to describe these as streams.

**A short road to death.**— We have seen, as in Fig. 2, that the appearance of streams remains pronounced as the particle energy is decreased. At sub-TeV energies, simulations become prohibitive, due to the decreasing  $e^\pm$  Larmor radius and the associated need for smaller time steps to accurately compute trajectories to determine the amount of time elapsed before propagation is isotropized. However, in Ref. [18] a scaling for the time of transition

to the diffusive regime for protons was found as

$$t_d \sim 10^4 \left( \frac{l_{\text{max}}}{150 \text{ pc}} \right)^\beta \left( \frac{1000 \text{ TeV}}{E} \right)^\gamma \left( \frac{B_{\text{rand}}}{4 \mu\text{G}} \right)^\gamma \text{ yr}, \quad (5)$$

where  $\beta \simeq 2$  and  $\gamma = 0.25 - 0.5$  for Kolmogorov (and similarly for other spectra).

As already noted, energy losses are a vital consideration for electrons and positrons, since the energy loss timescale for high-energy  $e^\pm$  due to synchrotron radiation and inverse-Compton scattering is only

$$t_l \sim 10^5 \left( \frac{1 \text{ TeV}}{E} \right) \left( \frac{5 \mu\text{G}}{B_{\text{tot}}} \right)^2 \left( \frac{1 \text{ eV cm}^{-3}}{\epsilon_\gamma} \right) \text{ yr}, \quad (6)$$

where  $\epsilon_\gamma$  is the ambient photon energy density. It is apparent that lower energy  $e^\pm$  than those considered in our simulations will be of interest. These do not experience the Klein-Nishina suppression of higher-energy backgrounds, so we include estimates for the infrared and optical backgrounds near Earth [35] with the CMB, giving  $\epsilon_\gamma \sim 1 \text{ eV cm}^{-3}$  (neglecting variations with energy in the IC cross section). We see, for  $l_{\text{max}} = 150\text{--}250 \text{ pc}$  and  $B_{\text{tot}} = 4\text{--}7.5 \mu\text{G}$ , that  $t_l = t_d$  for  $E_c \approx 10\text{--}1000 \text{ GeV}$ .

The implication is that  $e^\pm$  with  $E \gtrsim E_c$  are expected to lose their energy *prior to leaving streams and never reach the diffusive regime*. This would have profound consequences for observing  $e^\pm$  with energies exceeding  $E_c$  at Earth, since their densities are then governed by the structure of the local Galactic field. This is quite different than when dealing with protons, which experience negligible energy loss and arrive from much greater distances after diffusing through many field domains.

**What about  $B_{\text{reg}}$ ?**— We have also examined the effect of including an overall regular field component. In Fig. 3, we show the outcomes from setting  $B_{\text{reg}}/B_{\text{rand}} = 0, 1, \text{ and } 5$ , while keeping  $B_{\text{reg}} + B_{\text{rand}} = 3 \mu\text{G}$ . We have again used the source positioned at the center of Fig. 1, with  $E_g = 1 \text{ PeV}$ . It is clear that, rather than diminish streams, including a regular field accentuates them.

Results with a dominant random field, as for  $B_{\text{rand}} \sim 7.5 \mu\text{G}$  and  $B_{\text{reg}} \sim 3 \mu\text{G}$ , are close to a pure random case. For  $B_{\text{reg}} \approx B_{\text{rand}}$ , streams are aligned with neither the regular field nor the large-scale random components. In these scenarios, the direction of the nearby “regular” field likely lies off of the Galactic plane (as is observed).

For larger  $B_{\text{reg}}/B_{\text{rand}}$ , we see that propagation becomes ever more one dimensional, with the stream continuing to greater distances. We also find that the difference in  $D_{\parallel}$  compared to the phenomenological  $D$  vanishes. Even when we decrease  $l_{\text{max}}$  to 50 pc, for which Eqs. (5) & (6) imply a very large  $E_c$  for a purely random field, we see the same basic behavior for  $B_{\text{reg}}/B_{\text{rand}} = 5$  as in Fig. 3. Thus, in this regime the importance of streams would likely persist even at energies below the nominal  $E_c$  associated with the random component.

**Discussion and conclusions.**— For electrons and positrons at the energies of current and future interest, we have seen that a consideration of energy losses when propagation is treated as occurring in a turbulent magnetic field alone can significantly change the expectations for observations, particularly for nearby sources. One consequence is a limitation to the number of contributing sources at Earth. This holds when  $e^{\pm}$  propagation in the local Galactic magnetic field can be described using such a turbulent field coherent over a scale of  $\sim 100 \text{ pc}$ . We discuss some of the implications that follow.

*Cosmic-ray electrons:* As illustrated in Fig. 1, even for multiple local sources, at progressively higher energies one would be increasingly fortunate to be located in a stream. Crossed streams, where multiple sources would be inferred, are relatively rare. This is radically different than expected in isotropic models, where *all* nearby sources contribute to the flux and lead to a spectrum containing numerous features [16, 36]. However, the measured electron spectrum has a smooth variation [6].

These structures could be very important at the highest energies for  $e^{\pm}$ , since energy losses are severe, and fluxes may be present even at TeV–PeV energies. Several nearby pulsars are known to accelerate  $e^{\pm}$  to these energies [11]; however, since streams do not follow straight lines, enhancements could appear in any direction, even when the source position is known. High-energy electron and positron anisotropies would be more complex than a simple dipole oriented with the source, as in diffusive prescriptions (see, e.g., [16, 37]).

*Positrons (and dark matter):* Aside from the general rise associated with the positron excess [3], positron data are also rather featureless and have been interpreted in terms of a single pulsar. Indeed, the smaller number of

positron sources would imply fewer streams. One consequence could be an enhanced flux from an otherwise unremarkable source, easing concerns of exceeding the spin-down power of a lone pulsar. Limiting the sources that can contribute at Earth in this way could account for the smooth electron and positron spectra, since  $e^{\pm}$  fluxes from a source outside of a stream are diminished.

The closer propagation is to isotropy, the more likely a particular source contributes a finite flux. We have seen in Fig. 1 that such “clouds” are occasionally realized. One can determine volume filling factors as a function of energy, distance, and time. However, one could argue that the positron excess suggests that at least one source does reach Earth, so we leave such probabilistic arguments to elsewhere to keep focus on the larger points.

However, a scenario in which none of the local astrophysical positron sources actually reach Earth becomes a distinct possibility. At first glance, this would seem to require the diffuse injection of positrons, as from the smooth Galactic dark matter halo. However, positron models based on annihilations in dark matter substructure (e.g., [38–41]) are affected, since each is a continuously emitting source. While it is even less likely for the largest “clumps” to contribute, the subhalo mass function implies numerous less massive objects [42, 43]. Any of these could yield an apparent flux larger than expected from isotropic propagation (as assumed in, e.g., [44, 45]).

*Our Galactic neighborhood:* Streams would naturally limit the number of sources that contribute to the electron and positron fluxes at Earth, without fine tuning the fluxes from myriad sources to achieve the smoothness present in the measured spectra. They could also ease energetics concerns by concentrating the flux from a source. However, we have seen this to be a significant departure from the usual picture, and since  $E_c$  depends strongly on the actual parameters, which vary throughout the Galaxy, comparing to local data is essential.

The complex nearby environment of Earth [46] can be uniquely probed by  $e^{\pm}$  due to their limited lifetimes. The above simulations contain the common assumption of a static field with no feedback effects. It is likely that  $e^{\pm}$  propagation is more involved than this, and it is important to understand at what level. Depending on the interstellar medium, collective effects could increase scattering en route below some energy [47, 48], as proposed to explain breaks in cosmic-ray protons [49], or in the vicinity of sources [50–52]. We thus encourage their examination in a consistent framework relevant near Earth that includes energy losses. In any case, we have found the investigation of  $e^{\pm}$  to be richer than often assumed and open to further development.

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